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A NEW METHOD FOR THE DETERMINATION OF DEFLECTIONS OF THE VERTICAL FROM ASTROGEODETIC AND INERTIALLY DERIVED DATA*

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ABSTRACT: The paper first presents a review of prior developed methods for the determination of deflections of the vertical from astrogeodetic and inertially obtained data under consideration of gyro bias eliminations in Litton's local-level system. It then derives a Wiener-type optimal solution in semi-flat terrain under utilization of the collocation method in physical geodesy with prior emphasis on non-linear gyro bias removal and discusses the ramifications of presently and potentially available gyro and accelerometer hardware. Thereafter, a refined advanced method and deflection determination in strongly mountainous terrain is addressed under possible modification of the semi-flat terrain solutions and employment of spatial collocation. Finally, consideration is given to the associated problem of area adjustment solutions.

1. INTRODUCTION. The selective interpolation of deflections of the vertical by means of initial and terminal astrogeodetic, and inertial data has been pursued by the U.S. Army Engineer Topographic Laboratories (ETL) and the Geodetic Survey of Canada since about 1976. The inertial equipment employed has been the Rapid Geodetic Survey System (RGSS), developed by Litton Systems for ETL. Presently installed gyros, accelerometers, and velocity quantizers have permitted average deflection component accuracies of 1.5 arcsec rms for single runs of 60 km length or approximately 2 hours. The installment of available superior hardware in the horizontal channels of the RGSS and the utilization of improved post-mission data reduction methods is expected to result in average deflection component accuracies of about 0.4 arcsec rms for single runs. Network adjustments under consideration of data obtained from quasi-parallel runs may thus facilitate average deflection component accuracies of 0.3 arcsec rms. In this respect, it should be emphasized that the combination of new gyros and accelerometers with relatively small correlation times and improved data reduction methods is particularly effective. This paper reviews in the next section previously developed methods for the determination of deflections of the vertical. In the third section, a Wiener-type solution for

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semi-flat terrain is presented together with ramifications of present and improved gyro and accelerometer hardware. The fourth section outlines a refinement of the optimized method. The influence of mountainous terrain is considered in the fifth section, and the sixth section addresses the problem of adjustment solutions.

2. REVIEW OF PREVIOUSLY DEVELOPED METHODS: Baussus von Luetzow [1981a] presented in detail the approximate mathematical solution for deflections of the vertical as presently applied in the context of RGSS utilization. In this respect, the basic solution for the prime deflection of the vertical is

$$\xi_i = \left(\xi_o + \bar{\Phi}_{E_i} - \frac{\ddot{x}_i}{g}\right) + \frac{a_{N_i} - a_{N_o}}{g} - \frac{t_i}{t_n}\,\frac{a_{N_n} - a_{N_o}}{g} + \int_o^{t_i} (\gamma - \bar{\gamma})dt - \frac{t_i}{t_n}\int_o^{t_n} (\gamma - \bar{\gamma})dt \qquad (1)$$

In eq. (1), ξ is the prime deflection, $\bar{\Phi}_E$ is the systematic platform tilt about the east axis, $\ddot{x}$ is the static north accelerometer measurement, a_N is the correlated east accelerometer error, γ is the east axis angular drift rate error, $\bar{\gamma}$ is the associated constant gyro bias, g is normal gravity, and t is time. The subscripts refer to initial time t_o, time at the start of a vehicle stop with accelerometer reading t_i, and terminal mission time t_n:

Equation (1) may be supplemented by

$$\delta\xi_i = \delta\xi_o + \frac{t_i}{t_n}\left(\delta\xi_e - \delta\xi_o\right) - \left(\phi_{S_i} - \frac{t_i}{t_n}\phi_{S_n}\right) \qquad (2)$$

to account for astrogeodetic deflection errors and accelerometer bias errors. For a straight traverse, the last term in eq. (2) tends to cancel out.

The determination of $\overline{\Phi}_E$ and associated gyro bias functions is accomplished by means of the equations

$$\overline{\Phi}_E = t\left(\overline{\gamma} + \overline{\beta}\,\frac{\Omega_Z}{2}t - \overline{\alpha}\,\frac{\Omega_N}{2}t\right) \tag{3}$$

$$\overline{\Phi}_N = t\left(-\overline{\gamma}\,\frac{\Omega_Z}{2}\,t + \overline{\beta}\right) \tag{4}$$

$$\overline{\Phi}_Z = t\left(\overline{\gamma}\,\frac{\Omega_N}{2}\,t + \overline{\alpha}\right) \tag{5}$$

where $\overline{\Phi}_N$ is the systematic platform tilt about the north axis, $\overline{\Phi}_Z$ is the systematic azimuth platform attitude, $\overline{\alpha}$ is the constant azimuth axis angular drift rate bias, $\overline{\beta}$ is the constant north angular drift rate bias, Ω_N is the north earth rate, and Ω_Z is the vertical earth rate.

$$\overline{\Phi}_E(t_n) = \xi_{\text{estimated}}(t_n) - \xi_{\text{observed}}(t_n) \tag{6}$$

$$\overline{\Phi}_N(t_n) = \eta_{\text{estimated}}(t_n) - \eta_{\text{observed}}(t_n) \tag{7}$$

$$\overline{\Phi}_Z(t_n) = A_{\text{estimated}}(t_n) - A_{\text{observed}}(t_n) \tag{8}$$

It should be emphasized that the errors relating to the computation of $\overline{\Phi}_{E_i}$ and the static accelerometer reading in eq. (1) are reflected in the last four terms. Examination of equations (3) - (5) reveals that in moderate latitudes there exists relatively small coupling for time intervals not exceeding 2 hours. For this reason, linear approximations $\overline{\Phi}_E \approx \overline{\gamma}t$ and $\overline{\Phi}_N \approx \overline{\beta}t$ have been used with success. Highly accurate deflection determinations, accomplished by means of an advanced RGSS, would, however, require consideration of a terminal azimuth error, modified by a periodically applied Kalman filter correction.

The rms deflection error $\sigma_\xi(t_i)$ can be computed by covariance analysis involving the terms without parentheses in eq. (1). Under inclusion of the first two terms of eq. (2) it is

$$\text{var}\ \xi_i = \text{var}\ a_i + \text{var}\ \gamma_i + \left(1 - \frac{t_i}{t_n}\right)^2 \text{var}\ \xi_o + \left(\frac{t_i}{t_n}\right)^2 \text{var}\ \xi_n \tag{9}$$

where var a_i is the accelerometer-induced variance and var γ_i designates the gyro-induced variance.

The use of static accelerometer measurements in conjunction with initial and terminal deflections together with a simplified Kalman filter originated by Huddle [1977]. A weighted least-squares solution for deflections of the vertical was developed by Baussus von Luetzow [1978] under integration of the pertinent system of differential equations, constant travel intervals between vehicle stops, utilization of a limited number of deflection components together with collocation estimation, and velocity observations. In this approach, gyro biases were treated as random variables. For practical purposes, it is necessary to record the total velocity history or to drive the RGSS at a constant velocity.

3. <u>ADVANCED METHOD FOR SEMI-FLAT TERRAIN</u>. Since the determination of ξ_i according to eq. (1) is impaired because of the presence of four noise terms, a Wiener-type solution under consideration of all or of adjacent ξ_i-data in conjunction with a signal covariance function can be formulated as shown by Baussus von Luetzow [1981a]. For this purpose, eq. (1) under inclusion of the first two terms of eq. (2) may be formulated as

$$\xi_i = \hat{\xi}_i + n_i \tag{10}$$

where $\xi_i = \xi_o + \phi_{E_i} - \frac{\ddot{x}_i}{g}$ is a message variable, $\hat{\xi}_i$ is a signal variable, and the remaining terms denote a noise variable - n_i. The collocation method in physical geodesy in semi-flat terrain permits the estimation.

$$\hat{\xi}_e = \sum_i a_i(\hat{\xi}_i + n_i) = A_i(\hat{\Xi}_i + N_i) \tag{11}$$

where A_i is the matrix of regression coefficients a_i to be computed and $\hat{\Xi}_i$ is the corresponding $\hat{\xi}_i$ - matrix. It is then in matrix form, with bars indicating covariances,

$$\overline{\hat{\xi}_e \hat{\Xi}_k} = A_i(\overline{\hat{\Xi}_i \hat{\Xi}_k} + \overline{N_i N_k}), \quad \binom{i}{k} = 0, 1, \ldots n \tag{12}$$

The solution for the regression coefficient matrix follows as

$$A_i = \overline{\xi_e \hat{\Xi}_k} \left(\overline{\hat{\Xi}_i \hat{\Xi}_k} + \overline{N_i N_k} \right)^{-1} \tag{13}$$

The pertinent noise parameters applicable to the present RGSS are a standard deviation of 0.002° hr^{-1} and a correlation time of 3 hours for the G-300 gyros and a standard deviation of 10 mgal and a correlation time of 40 minutes for the A-200 accelerometers. Due to the sizable correlation times, the advanced method does not provide significantly better deflection estimates than the basic method. Considerably improved estimates would, however, result under utilization of intermediate deflection constraints. An advanced RGSS, with a standard deviation of 0.0002° hr^{-1} and a correlation time of about 5 minutes for G300-G2 gyros and a standard deviation of 1 mgal and an approximate correlation time of 5 minutes, would generate data commensurate with the potential of the advanced estimation method. In addition, it constitutes a statistical framework for an area adjustment under utilization of data relating to several traverses.

4. REFINEMENT OF ADVANCED METHOD. The basic method for the determination of deflections of the vertical, presented in section 2, does not consider the interaction of the two horizontal error differential equations of motion and the three gyro error differential equations. The latter are employed for the determination of constant gyro biases. For short traverse times and in connection with present RGSS data the basic method is certainly adequate. More accurate deflection determination by means of an advanced RGSS may require a refinement of the advanced method. This is consistent with observations made by Schwarz [1980].

A full refinement of the advanced method requires the integration of the whole system of differential equations and the application of RGSS Kalman filter corrections at stops, preferably also extended to Φ_z. For practical purposes, vehicle stop intervals should be 3 minutes and the vehicle speed should be approximately constant, preferably about 30 km hr^{-1}. As a result, ξ_i in eq. (1) would be replaced by

$$\xi_{ri} = \xi_i + C_i + \sum_{\nu} a_{i\nu}\xi_{\nu} + \sum_{\nu} b_{i\nu}\eta_{\nu} - n_i^{(2)} \tag{14}$$

where C_i denotes a systematic correction, $a_{i\nu}$ and $b_{i\nu}$ are weight factors obtained by numerical integration under consideration of Kalman filter corrections, and $n_i^{(2)}$ is a composite representation of modified gyro and accelerometer errors. Further, ξ_{ν} and η_{ν} indicate $\xi(t_{\nu})$ and $\eta(t_{\nu})$, respectively. Equation (14) may be formulated as

$$\hat{L}_i = \hat{\xi}_i - \left(\sum_{\nu} a_{i\nu}\hat{\xi}_{\nu} + \sum_{\nu} b_{i\nu}\hat{\eta}_{\nu}\right) = \left(\xi_o + \overline{\Phi}_{Ei} - \frac{\ddot{x}_i}{g} + C_i\right) - n_i^{(2)} \tag{15}$$

The message estimator is then

$$L_i = \hat{L}_i + n_i^{(2)} = \left(\xi_o + \overline{\Phi}_{Ei} - \frac{\ddot{x}_i}{g} + C_i\right) \tag{16}$$

In analogy with equation (11), any signal variable may be estimated according to

$$\hat{\xi}_e = \sum_i b_i(\hat{L}_i + n_i^{(2)}) = B_i \left\{ [L_i] + N_i^{(2)} \right\} \tag{17}$$

where B_i is the matrix of regression coefficients and $[L_i]$ and $N_i^{(2)}$ are corresponding signal and noise matrices. In accordance with eq. (13), the solution for the regression coefficient matrix follows as

$$B_i = \overline{\hat{\xi}_e[\hat{L}_k]} \left\{ \overline{[\hat{L}_i]\ [\hat{L}_k]} + \overline{N_i^{(2)}N_k^{(2)}} \right\}^{-1} \tag{18}$$

In general, eq. (18) requires the coordinates $x(t_{\nu})$, $y(t_{\nu})$, x_i, and y_i as computer program inputs.

It is apparent from the above analysis that the basic deflection estimation method is associated with correlated hardware errors and correlated errors due to the omission of linear aggregate deflection terms. These latter errors are not independent from one traverse to an adjacent one.

5. CONSIDERATION OF MOUNTAINOUS TERRAIN. The use of advanced or Wiener-type methods for deflection determination in semi-flat terrain requires a modification in mountainous terrain in order to compute signal covariances. As shown by Baussus von Luetzow [1981b], it is possible to represent deflections in mountainous terrain in the form

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \begin{Bmatrix} \tilde{\xi} \\ \tilde{\eta} \end{Bmatrix} + \begin{Bmatrix} \delta\xi_t \\ \delta\eta_t \end{Bmatrix} \tag{19}$$

where $\delta\xi_t$ and $\delta\eta_t$ are computable topographic "noise" terms, statistically non-stationary in character. It is, therefore necessary in connection with the utilization of an advanced RGSS to employ the transformation

$$\begin{Bmatrix} \xi_i^{(2)} \\ \eta_i^{(2)} \end{Bmatrix} = \begin{Bmatrix} \xi_i - \delta\xi_{ti} \\ \eta_i - \delta\eta_{ti} \end{Bmatrix} \tag{20}$$

in the advanced methods discussed in sections 3 and 4. This results in the modification of measured "message" information.

Following these data modifications, spatial collocation may be employed instead of planar covariance functions, also outlined by Baussus von Luetzow [1981b]. After completion of the signal estimation of $\hat{\xi}_e^{(2)}$, its corresponding "message" value is

$$\xi_e^{(2)} = \hat{\xi}_e + \delta\xi_{te} \tag{21}$$

Based on the above, the optimal estimation of deflections of the vertical from astrogeodetic and inertial data is not possible without a considerable computational effort. For this reason, the basic method presented in section 2 appears to be sufficient if data from one or two repetitive runs are employed. Regardless of these repetitions, the deflections accuracy achieved thereby will be somewhat impaired.

6. AREA ADJUSTMENT SOLUTIONS. The advanced methods presented in sections 4 and 5 simultaneously provide for area adjustment solutions. To assure quasi-uniform coverage and to avoid computational complexity and the influence of accelerometer scale factor variations, the traverses should be quasi-parallel and system calibration should be attempted prior to each run to eliminate correlations between systems errors referring to different traverses. Cross-traverse coverage would be beneficial as to data comparison at intersection points and for additional error reduction. Quasi-rectangular coverage with astrogeodetic data on the boundary would provide for economic survey extensions. For practical purposes, approximately 50 adjacent "message" data need to be used for a specific minimum error variance determination. Computer program inputs are bias-corrected "message" deflections, boundary deflections, their coordinates, and systems error variances and covariances associated with signal covariances. The refined advanced method requires a considerably greater programming and input effort. Because of the reasonable restriction to about 50 "message" data, it is not necessary to employ mixed data, i.e., ξ-, η-, and gravity anomaly data. Cross-covariances involving these variables are insignificant for shorter distances. Further, the presence of gyro heading sensitivity errors, reduced or unreduced, does not warrant undue complexity. As shown in section 5, solutions affected by formidable mountainous terrain require special consideration or will be impaired, respectively.

An inverse-space domain smoothing involving the use of orthogonal functions and deflection and gravity anomaly data within a large, flat, rectangular area has been developed by Bose [1980]. In the context of this method, measurement noise is assumed to be a zero mean uncorrelated process. Apart from this approximation, the "message" data has to be generated in a regular pattern which is often not possible. In order to achieve an effective smoothing under avoidance of Gibbs-type fluctuations, it is necessary to analytically approximate measurements for each elementary rectangle for the computation of coefficients $\hat{\alpha}_{mn}$ in the case of large m and n.

The Wiener-type solutions under utilization of approximate signal covariance functions and non-stationary, partially correlated system errors and the Bose solution have, therefore, advantages and disadvantages. The former ones are certainly more versatile and in principle minimum variance estimates, and the refined Wiener-type solution offers an additional advantage.

7. CONCLUSION: Advanced, Wiener-type filtering methods, including refined methods and consideration of computable topographic "noise" in mountainous areas, presented in this paper, will provide minimum variance estimates of deflections of the vertical from astrogeodetic and discrete, inertially derived data both for single and multiple traverses. Additional research, to include numerical integration of a system of differential equations and Kalman filter corrections, is necessary to facilitate a quantitative comparison between the method of section 3 and the refined method of section 4.

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